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Title: Time-Periodic Solutions for the Euler Equations: Rigidity, Flexibility, and Inverse Problems

Abstract: In this mini-course, I will present several aspects of time-periodic solutions for the incompressible Euler equations, a topic that has seen significant developments over the past decade. Our aim is to highlight both rigidity and flexibility phenomena, as well as their connections to inverse problems and analytical techniques.

We begin with the study of rigid structures, focusing in particular on rotating solutions. In the setting of vortex patches, rigidity is closely related to inverse problems, where one seeks to recover geometric or dynamical properties of the flow from partial information. These problems can be formulated as free boundary problems and analyzed using elliptic techniques, including variational methods and sharp isoperimetric inequalities of Talenti type. Another facet of rigidity will be explored through tools from complex analysis, such as conformal mappings and Faber polynomials.

In contrast, the flexibility aspect of the theory is illustrated through bifurcation phenomena. In particular, we discuss the existence of nontrivial time-periodic solutions bifurcating from steady states via perturbative methods, with special emphasis on the Crandall–Rabinowitz theorem.

We conclude with a discussion on the use of KAM theory to construct non-rigid time-periodic solutions.